A Binary, Non-convex, Variable-capacitated Supply Chain Model

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Abstract: **This paper is concerned with three-echelon supply chain design where each supplier provides a unique set of goods from known, possibly multiple, locations and where each outlet has a fixed, known demand so that it exhibits the features of the supply chain of an existing company that operates across Canada and in the United States of America (35 suppliers, 83 potential DC locations and 2,976 outlets). A mathematical model is presented whose solution determines the location and capacity level of Distribution Centers (DCs) and assigns outlets to the selected DCs. The model is unique in that it allows true variability in the choice of capacity level and so avoids the need to determine, a priori, a set of potential capacity levels. The design objective is to minimize fixed and variable costs for operations and transportation that account for decreasing marginal costs and economies of scale. This makes the model a binary, non-convex optimization problem. A piecewise linear approximation to the concave cost functions that captures the concept of "technology break-points" results in a model for which LINGO can quickly determine high quality solutions.**

Key Words: *Variable-capacitated; Supply chain design; Binary nonlinear concave program; Piecewise linearization*

1. Introduction

The optimal design of supply chains is critical in order for a company to become and / or remain globally competitive. Excellent reviews of the published literature can be found in, for example, [1]-[4]. The decision phases for supply chain management can be classified as strategic, tactical and operational [5]. Strategic management is concerned with decisions such as the number, location and the capacity level of Distributions Centers (DCs). Tactical decisions include the determination of the flow of products between the supply chain echelons over a specific planning horizon.

Operational decisions involve items such as truck routing. This paper addresses both strategic and tactical decisions for a three echelon supply chain with suppliers, distribution centers (DCs), also called plants and outlets, also called customers.

The Simple Plant Location Problem (SPLP), also known as the Uncapacitated Facility Location (UFL) problem, given in [6], [7] has binary variables to indicate whether or not a plant is built at a pre-selected location and has fractional variables to determine the percentage of demand for each customer met by a particular plant. The cost functions include transportation costs and fixed cost for building. While SPLP is NP-hard, primal-dual algorithms can be effective in solving large SPLP problems to optimality, or near optimality [8]. Generalizations to the SPLP include the consideration of trade-offs between inventory, transportation and building costs [9], an ability to choose the optimal mix between dedicated and flexible technologies [10], the inclusion of costs modeled by the convex part of a non-linear, increasing function [11], the extension to two echelon supply chains [12] and multi product supply chain designs [13], [14]. An exact solution method when there are convex transportation costs can be found in [15].

In [16] a new formulation is given for the multi-level supply chain network design and is shown that small and medium sized instances of this formulation can be solved by, for example, CPLEX. A mixed integer linear programming model, along with a Lagrangean based solution procedure, is given in [17] for three-echelon multi-capacitated supply chain network design. The capacity levels and locations of the DCs and suppliers are chosen from predetermined locations and from finite sets of discrete capacity levels, respectively. The cost functions in the model include the variable costs associated with the chosen capacity level. In [18], the variable costs are charged only for the level of activity resulting in a binary quadratic optimization model. Beasley [19] presented a Lagrangian-based solution procedure for different location problems including p – median and uncapacitated location

International Journal of Supply Chain Management
IJSCM, ISSN: 2050-7399 (Online), 2051-3771 (Print)

problem. The capacitated plant location problem for offshore oil exploration platforms is studied in [20].

This paper is concerned with three-echelon variable-capacitated supply chain network design with modifications to capture some features of a model company with operations across Canada and in the United States of America. (The identity of the company is protected by a non-disclosure agreement.) The company's supply chain network has a fixed set of 35 suppliers, each providing a unique set of goods, some from multiple locations. The location of the built DCs will be chosen from a pre-selected, finite set of 83 potential DC locations. Once built, the DC will receive goods from the closest supplier. Consequently, the supplier network influences the location and capacity of the DCs only through the in-bound transportation costs. It is assumed that the suppliers have sufficient capacity to meet total network demand. The 2,976 outlets, organized into 133 clusters according to the first two digits of their postal code, receive regular shipments of a variety of goods, in varying quantities. To reduce the complexity of the problem historical data is used to create a *generic out-bound pallet* and to determine a fixed demand for the generic pallet for all outlets. Further, historical transportation costs can be used to determine an average out-bound cost per pallet per kilometer. Assuming that a particular supplier uses identical pallets from all of its locations an average *in-bound* cost per pallet per kilometer can be determined. The in-bound pallets are different for different suppliers and are different than the out-bound pallet. The use of the cost per pallet-kilometer factors avoids the need to consider truck routes, truck types, loads and driver schedules.

The decision variables for the supply chain design problem considered in this paper will be the selection of DC location, DC capacities and outlet to DC assignment. The DC locations are chosen from a predetermined set of potential DC locations but the capacity level will be determined by the total demand from the assigned outlets. Consequently, the model allows true variability in capacity selection and the operational costs will not include costs for unused capacity.

The supply chain design will be driven by the objective of minimizing the in-bound transportation costs, out-bound transportation costs, fixed DC set up costs, building costs, and operational costs. The transportation costs will be linear functions of the number of in-bound and out-bound pallets shipped. The building and operational costs will be modeled to include economies of scale and decreasing marginal costs. Consequently, these cost functions are concave, which, together with the fixed DC set up cost, will result in the establishment of fewer DCs with larger capacity levels. This feature supports the consolidation policy in supply chain design.

The Binary, Non-Linear, Concave Optimization (BO) model is developed and demonstrated in section 2. To capture the concept of "technology break-points" a piecewise linearization (PWL) of the BO model is given in section 3. Numerical examples abstracted from the model company are given to demonstrate the effectiveness of LINGO to solve the supply chain design models. The final example has 11122 variables and 216 constraints and its solution validates the applicability of the model.

2. Model Formulation

This paper adopts much of the notation and model development as in [18]. The outlets, potential DC locations, and suppliers are indexed by $r \in R$, $d \in D$, and $s \in S$, respectively. The binary variables x_d and y_{dr} indicate if a distribution center is to be built at location d and if distribution center d is to provision outlet r, respectively. Since y_{dr} is binary,

$$\sum_{d\in D} y_{dr} = 1 \qquad \forall\, r \in R, \tag{1}$$

ensures that each outlet is assigned to a single DC. Let p_r be the out-bound pallet demand at outlet r. Since x_d is binary,

$$\sum_{r\in R} p_r\, y_{dr} - M x_d \leq 0 \qquad \forall\, d \in D, \tag{2}$$

ensures that, for each DC, the total demand from the outlets assigned to that DC does not exceed the maximum allowable capacity level M, where, for example, M could be set as the total network demand.
Clearly, the model allows complete variability, from 0 to M, in the selection of the capacity level for each DC. In fact, the model sets the capacity level of a DC to the total demand from its assigned outlets. This avoids the need to make a priori capacity level selections.
Let k_{dr} be the distance in kilometers from distribution center d to outlet r. If ω is the cost of shipping one out-bound pallet one kilometer, then the total out-bound transportation cost is

$$T_{out}(y) = \omega \sum_{d\in D}\sum_{r\in R} k_{dr}\, p_r\, y_{dr}. \tag{3}$$

Let $\hat{k}_{sd}$ be the distance in kilometers from the nearest location of supplier s to distribution center d, and let ω_s be the cost to ship one in-bound pallet from supplier s one kilometer. Let ρ_s be the percentage of the generic out-bound pallet that is provided by supplier s. Then $(\rho_s p_r)$ is the number of in-bound pallets from supplier r required to assemble the out-bound pallets delivered to outlet r. The total in-bound transportation cost is

$$T_{in}(y) = \sum_{s\in S} \omega_s \sum_{d\in D}\sum_{r\in R} \hat{k}_{sd}\,(\rho_s p_r) y_{dr}$$

$$= \sum_{s\in S} \omega_s\, \rho_s \sum_{d\in D} \hat{k}_{sd} \sum_{r\in R} p_r\, y_{dr}. \tag{4}$$

To capture economies of scale the operational (variable) costs are modeled with the nonlinear function

$$V(y) = \sum_{d \in D} \alpha \left(\sum_{r \epsilon R} p_r \, y_{dr} \right)^{\beta} \qquad (5)$$

where $\alpha > 0$ and $0 < \beta < 1$. While α and β are DC independent, the operational cost of each DC must be calculated separately because the DCs have unique capacity levels. The benefit is that the variable cost is only charged for the actual activity level in a DC and not for the excess capacity that results if DCs are selected from a finite set of pre-specified capacity levels. Similarly, the building costs with

$$B(y) = \sum_{d \in D} \gamma \left(\sum_{r \epsilon R} p_r \, y_{dr} \right)^{\delta} \qquad (6)$$

where $\gamma > 0$ and $0 < \delta < 1$. By using α and β for all operational costs and γ and δ for all land and building costs, the assumption is that costs are independent of location and this is likely not the case. It would be a simple matter to subscript the cost function parameters in order to account for location and this would not change the complexity of the model. In practice, the cost parameters can be approximated using regression with historical cost data.

In order to produce designs with fewer DCs the fixed DC set up cost

$$F(x) = F \sum_{d \in D} x_d, \qquad (7)$$

where F is a positive parameter, is included in the objective function.

Combining the above, results in the following binary optimization model with linear constraints and a concave objective function.

$Min \qquad f(x,y) = T_{out}(y) + T_{in}(y) + V(y) + B(y) + F(x) \quad (\boldsymbol{BO})$

Subject to:

$$\sum_{d \in D} y_{dr} = 1 \qquad \forall\, r \in R$$

$$\sum_{r \in R} p_r \, y_{dr} - M x_d \le 0 \qquad \forall\, d \in D$$

$$x_d \in \{0,1\} \qquad \forall\, d \in D$$

$$y_{dr} \in \{0,1\} \qquad \forall\, d \in D, \; r \in R$$

The model is tested on nine examples abstracted from data available from the model company and described in table 1. The cost function parameters used are $\alpha = 256.03$, $\beta = 0.7706$, $\gamma = 519.18$, $\delta = 0.5978$ and $F = 50{,}000$. The maximum capacity level is set at $M = 10{,}000$.

Table 1. Description of the test problems

Ex.	Total Network Pallet Demand	Number of Outlets	Number of Potential DCs	Number of Variables	Number of Constraints
1	1,107	11	10	120	21
2	1,663	10	5	55	15
3	725	8	5	45	13
4	592	9	3	30	12
5	523	6	3	21	9
6	19,924	18	14	266	32
7	16,726	15	12	192	27
8	27,019	10	7	77	17
9	29,362	8	5	45	13

The global solver in LINGO 14 was run on a 64-bit DELL PC with two 2.50 GHz threads (cores) and with 32 GBs of RAM. All problems were solved to optimality and the results in table 2 show the solution time in seconds, the indexes of the selected DCs and the corresponding capacity levels, and the optimal objective function value. The excessive time required to solve examples 6 - 8 is one motivation to consider a piece-wise linearization of the concave objective function. Another motivation is that the break-points in the piece-wise linearization can capture technology break-points. While the cost functions are linear, the slope of the linear function decreases at the break-points corresponding to a decreased cost per pallet with a higher level of technology.

Table 2. Solution statistics for the BO model

Ex.	Time (sec.)	Selected DC IDs	Capacity of Selected DCs	Optimal $f(x,y)$
1	29	1	1,107	824,690
2	3	1	1,663	1,065,610
3	1	1	725	337,481
4	1	3	592	259,650
5	1	2	523	342,340
6	145,011	1 8 13	5,934 4,013 9,977	12,012,173
7	985	1 12	6,756 9,970	10,183,400
8	459	2 4 7	9,208 7,845 9,966	17,609,700
9	31	2 3 4 5	1,777 8,233 9,937 9,415	19,489,200

3. Piece-Wise Linearization

The concave parts of the objective function are replaced with piece-wise linearizations. That is, $f(x, y)$ is replaced by

$$f_{pwl}(x, y) = T_{out}(y) + T_{in}(y) + V_{pwl}(y) + B_{pwl}(y) + F(x) \qquad (8)$$

where $V_{pwl}(y)$ is obtained as follows. ($B_{pwl}(y)$ is obtained in an analogous way.) Define the break-points Δ_i, $i = 0, 1, 2, 3, 4, 5$ where

$$0 < \Delta_0 < \Delta_1 < \Delta_2 < \Delta_3 < \Delta_4 < \Delta_5 = M \qquad (9)$$

and denote the function values at the break-points by

$$V_i = \alpha\,(\Delta_i)^{\beta}. \qquad (10)$$

Define, for $d \in D$,

$$\hat{y}_d = \sum_{r \in R} y_{dr}\, p_r. \qquad (11)$$

Then,

$$V_{pwl}(\hat{y}_d) =$$

$$\begin{cases} V_0 + (\hat{y}_d - \Delta_0)\left(\frac{V_1 - V_0}{\Delta_1 - \Delta_0}\right), & if \quad \Delta_0 \le \hat{y}_d \le \Delta_1 \\ V_1 + (\hat{y}_d - \Delta_1)\left(\frac{V_2 - V_1}{\Delta_2 - \Delta_1}\right), & if \quad \Delta_1 \le \hat{y}_d \le \Delta_2 \\ V_2 + (\hat{y}_d - \Delta_2)\left(\frac{V_3 - V_2}{\Delta_3 - \Delta_2}\right), & if \quad \Delta_2 \le \hat{y}_d \le \Delta_3 \\ V_3 + (\hat{y}_d - \Delta_3)\left(\frac{V_4 - V_3}{\Delta_4 - \Delta_3}\right), & if \quad \Delta_3 \le \hat{y}_d \le \Delta_4 \\ V_4 + (\hat{y}_d - \Delta_4)\left(\frac{V_5 - V_4}{\Delta_5 - \Delta_4}\right), & if \quad \Delta_4 \le \hat{y}_d \le \Delta_5 \end{cases} \qquad (12)$$

and,

$$V_{pwl}(y) = \sum_{d \in D} V_{pwl}\,(\hat{y}_d). \qquad (13)$$

The five break-points $\Delta_1 = 750$, $\Delta_2 = 1650$, $\Delta_3 = 2900$, $\Delta_4 = 5000$ and $\Delta_5 = 10000$, was motivated by the set of five discrete capacity levels used by the model company. The model was also tested using the five evenly distributed break-points $\Delta_1 = 2000$, $\Delta_2 = 4000$, $\Delta_3 = 6000$, $\Delta_4 = 8000$ and $\Delta_5 = 10000$.

The BO model, and the PWL with both sets of break-points, produced identical solutions, i.e., they selected the same DCs with the same capacity level. What is of interest is the time required to find the solution. The times are summarized in table 3.

Table 3. Solution times in seconds

Ex.	BO	PWL (even)	PWL (Selected)	Best Time
1	29	1	2	1
2	3	1	1	1
3	1	1	1	1
4	1	1	1	1
5	1	1	1	1
6	145,011	868	3,266	868
7	985	286	151	151
8	459	186	142	142
9	31	21	6	6

In terms of solution times, the PWL model is superior to the BO model. Except for examples 1 and 6, the PWL model with selected break-points is superior to that with evenly distributed break-points. However, the number and values of the break-points should be determined by balancing the desire for fewer breakpoints against a better piecewise linear approximation and by actual technology improvement levels.

Observing that the PWL model with selected break-points is the superior model, it is applied to the model company's complete supply chain. After 210,477 seconds the best, non-optimal, objective function produced by LINGO for the BO model was $f(x, y) = 8,747,190$. LINGO solved the PWL model to optimality after 2,330 seconds and it gave a feasible solution to the BO model with $f(x, y) = 4,432,050$. The PWL model produced a solution that was 49% less costly in time that was two orders of magnitude smaller.

4. Conclusion

A binary, nonlinear, concave optimization model was developed for three-echelon, supply chain network design with true variability in capacity selection for the Distributions Centers and with cost functions that captured decreasing marginal costs and economies of scale. A piece-wise linearization of the objective function was introduced to both improve solution times and to capture technology break-points (related to economies of scale). It was shown that, for the model company's complete supply chain, the PWL model produced a higher quality solution than the BO model in time lower by orders of magnitude, when solved by LINGO.

Acknowledgments

We thank the model company for providing a model of their supply chain operations. This work was supported by the Natural Sciences and Engineering Research Council of Canada under grant numbers 436581-12 and 1798-03.

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